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Fuzzy Criteria for the Susceptible, Infected and Recovered Model with Specific Reference to Smallpox, Measles under: A Review

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Abstract—*In this manuscript an attempt has been taken to look at the SIR model for the mathematical modeling of diseases. We will discuss the mathematics behind the model and various tools for judging effectiveness of policies and control methods. We will complete the work with an example using the infectious disease Varicella, commonly known as the Chicken Pox.*

Keywords: *Mathematical modeling, Fuzzy Criteria, Review.*

1. INTRODUCTION

In order to model the progress of an epidemic in a large population, comprising many different individuals in various fields, the population diversity must be reduced to a few key characteristics which are relevant to the infection under consideration. For example, for most common childhood diseases that confer long-lasting immunity it makes sense to divide the population into those who are susceptible to the disease, those who are infected and those who have recovered and are immune. These subdivisions of the population are called **compartments**.

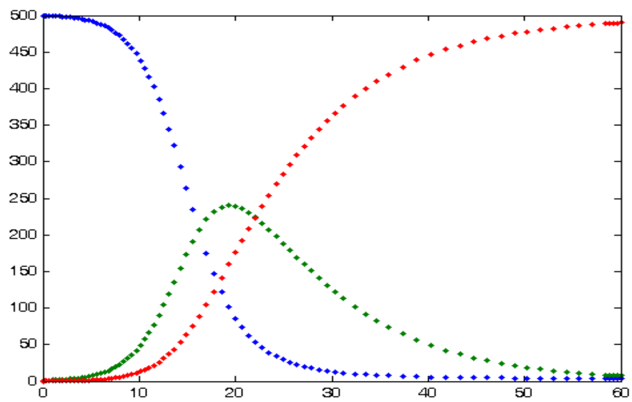
1.1 The Sir Model

Standard convention labels these three compartments S (for susceptible), I (for infectious) and R (for recovered). Therefore, this model is called the SIR model. This is a good and simple model for many infectious diseases including measles, mumps and rubella.

The letters also represent the number of people in each compartment at a particular time. To indicate that the numbers might vary over time (even if the total population size remains constant), we make the precise numbers a function of t (time): $S(t)$, $I(t)$ and $R(t)$. For a specific disease in a specific population, these functions may be worked out in order to predict possible outbreaks and bring them under control.

1.2 The SIR model is dynamic in three senses

As implied by the variable function of t, the model is dynamic in that the numbers in each compartment may fluctuate over time. The importance of this dynamic aspect is most obvious in an endemic disease with a short infectious period, such as measles in the UK prior to the introduction of a vaccine in 1968. Such diseases tend to occur in cycles of outbreaks due to the variation in number of susceptible ($S(t)$) over time. During an epidemic, the number of susceptible individuals falls rapidly as more of them are infected and thus enter the infectious and recovered compartments. The disease cannot break out again until the number of susceptible has built back up as a result of babies being born into the susceptible compartment.



Blue=Susceptible, Green=Infected, and Red=Recovered.

1.3 Bio-mathematical deterministic treatment of the SIR model

1.4.1 The SIR model without vital dynamics

A single epidemic outbreak is usually far more rapid than the vital dynamics of a population, thus, if the aim is to study the immediate consequences of a single epidemic, one may neglect the birth-death processes. In this case the SIR system described above can be expressed by the following set of ordinary differential equations:

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

This model was for the first time proposed by O. Kermack and Anderson Gray McKendrick, who had worked with the Nobel Laureate Ronald Ross.

This system is non-linear, and does not admit a generic analytic solution. Nevertheless, significant results can be derived analytically.

Firstly note that from:

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0 \quad \text{..................(1.1)}$$

It followa that:

$$S(t) + I(t) + R(t) = cons\tan t = N$$

expressing in mathematical terms the constancy of population N. Note that the above relationship implies that one can study the equation for only two of the three variables.

Secondly, we note that the dynamics of the infectious class depends on the following ratio:

$$R_0 = N\frac{\beta}{\gamma}$$

the so-called basic reproduction number (also called basic reproduction ratio). This ratio is derived as the expected number of new infections from a single infection in a population where all subjects are susceptible

2. MATHEMATICAL MODELS

A **mathematical model** is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed **mathematical modeling**. Mathematical models are used not only in the natural sciences (such as physics, biology, earth science, meteorology) and engineering disciplines (e.g. computer science, artificial intelligence), but also in the social sciences (such as economics, psychology, sociology and political science);

2.1 Examples of mathematical models:

1: Many everyday activities carried out without a thought are uses of mathematical models. A geographical map projection of a region of the earth onto a small, plane surface is a model which can be used for many purposes such as planning travel.

2: Another simple activity is predicting the position of a vehicle from its initial position, direction and speed of travel, using the equation that distance travelled is the product of time and speed. This is known as dead reckoning when used more formally. Mathematical modeling in this way does not necessarily require formal mathematics; animals have been shown to use dead reckoning.

3: *Population Growth*. A simple (though approximate) model of population growth is the Malthusian growth model. A slightly more realistic and largely used population growth model is the logistic function, and its extensions.

4: *Neighbor-sensing model* explains the mushroom formation from the initially chaotic fungal network.

5: *Computer Science*: models in Computer Networks, data models, surface model.

6: *Mechanics*: movement of rocket model Modeling requires selecting and identifying relevant aspects of a situation in the real world.

2.5 Building blocks

There are six basic groups of variables namely: decision variables, input variables, state variables, exogenous variables, random variables, and output variables. Since there can be many variables of each type, the variables are generally represented by vectors.

Decision variables are sometimes known as independent variables. Exogenous variables are sometimes known as parameters or constants.

3. FUZZY LOGIC AND MEASLES VACCINATION: DESIGNING A CONTROL STRATEGY:

3.1 Methods:

Fuzzy Decision Making techniques are applied to the design of the vaccination compaign.

3.2 Essentials of Fuzzy Logic:

Fuzzy Logic is a superset of conventional (Boolean) logic that has been developed to handle the concept of partial truth-truth values between ‘completely true’ and ‘completely false’. It may be considered a powerful tool for dealing with imprecision, uncertainty and partial truth aiming at tractability, robustness and low-cost solutions for real-world problems. Fuzzy Logic was formally established by Lotf-Zadeh, in 1965, with the introduction of the concept of membership degree , accord ing to which a set could have elements that belong partially to it. In this way, if one assumes that X is the Universe set, the Fuzzy subset A of the X has a characteristic function associated to it.

$$\mu_A : X \to [0,1] \quad \text{………………………………….(3.1)}$$

that is generally called membership function. The idea is that for each $x \in X, \mu_A(X)$ indicates the degree to which x belongs to the fuzzy subset A. In this sense, Fuzzy Logic differs from Crisp Logic (i.e. classical binary logic) basically for allowing that its statements assume other values besides false and true . Therefore, Fuzzy Logic concernsa logical system which is much closer in spirit to human thinking and natural languages than the traditional logic system.

First, the original theory of fuzzy sets was formulated in terms of the following specific operators of sets: union, intersection and complement. These set theoretic operations for fuzzy sets are defined via their membership functions and mathematical operations like max (maximum) and min(minimum). In accordance with this, if A and B are two fuzzy sets, the membership function $\mu_{A\cup B}$ of the union $A \cup B$ is defined for all $u \in U$ by

$$\mu_{A\cup B}(u) = \max\{\mu_A(u), \mu_B(u)\} \quad \text{……………..(3.2)}$$

The membership function $\mu_{A\cap B}$ of the intersection $A \cap B$ is defined for all $u \in U$ by

$$\mu_{A\cap B}(u) = \min\{\mu_A(u), \mu_B(u)\} \quad \text{……………………..(3.3)}$$

Finally, the membership function $\mu_{\bar{A}}$ of the Complement of a fuzzy set A is defined for all $u \in U$ by

$$\mu_{\bar{A}}(u) = 1 - \mu_A(u) \quad \text{……………………….(4)}$$

3.3 Fuzzy Logic in medicine:

Nowhere in the field of science is the need for tools to deal with uncertainty more critical than in medicine and biology. Disease diagnosis involves several levels of imprecision and uncertainty particularly in epidemiological studies. A single disease may manifest itself quite differently in different patients and with different disease status. Further, a single symptom may be indicative of different diseases, and the presence of several diseases in a single patient may disrupt the expected symptom pattern of any of them. This may cause a tremendous amount of imprecision and uncertainty in the interpretation of effect measures in analysis. Also, the best and most helpful descriptions of disease entities often use linguistic terms that are inevitably vague.

3.4 Fuzzy Decision Making:

Making a decision is one of the most fundamental activities of human being This is particularly true in public health where decisions usually have relevance for millions of people. In the field of vaccination strategy design, decision making concerning the target population for the immunization programme the proportion of susceptibles to be vaccinated, the optimal age at which to immunize children and the nature of the strategy e.g. selective or indiscriminate, are examples of the variables to be optimized, subject to a set of constraints.

In their first paper on Fuzzy Decision Making, Bellman and zadeh suggest a fuzzy model of decision making in which relevant goals and constraints are expressed in terms of fuzzy sets, and a decision is determined by an appropriate aggregation these fuzzy sets. The decision models have the following components.

a set A of possible actions

a set of goals $G_i(i \in N)$), each of which is expressed in terms of a fuzzy set defined on A ;

a set of constraints ,$C_j(j \in M)$ each of which is also expressed in terms of a fuzzy set defined on A .

The fuzzy set of decision, D, is that which simultaneously satisfies the given goals G_i and constraints C_j and is

$$D(a) = \min[\inf_{i\in N} G_i(a), \inf_{j\in M} C_j(a)]$$

For all a belong toA.

3.5 Designing the vaccination strategy:

We assume the objective of a vaccination campaign to be the reduction in measles infection of children under 14 years the age interval where the measles virus is most likely to be circulating. This assumption is based on previous works which demonstrated that the force of infection of the measles virus has a strong age-dependence, peaking around 2 years of age in the absence of vaccination Therefore, in spite of the high proportion of cases aged 20 years, the highest incidence rate(normalized per 100 000 inhabitants) observed during the epidemic occurred in children under 5 years. In addition, contact patterns suggest that adult cases are the product of infective contacts of susceptible individuals in that age group with children under 14 years .the target age interval of the vaccination campaign. All subsequent analysis in this work is based on these assumptions.

We begin by considering eight possible vaccination strategies, comprised of combinations of selective vaccinations, (S), meaning vaccinating only children without past vaccination records, and indiscriminate vaccination, (I), i.e. vaccinating children irrespective of previous immunization history (i and j stand for the age intervals). Besides, we considered the use of mobile units (MU), meaning those vaccination sites that are not part of the primary care network, as opposed to fixed units (FU), those belonging to the network. Table 1 shows the various vaccination strategies considered. The number of children, as well as the estimated proportion and number of susceptible children in each age interval of Allahabad district are shown in Table 2 is the maximum theoretical number of children to be vaccinated in each age group in order to

Table 1: Possible vaccination strategies

1 - $S_{9m-6y} - I_{6y-14y,MU+FU}$
2- $S_{9m-6y} - I_{6y-14y,FU}$
3- $S_{9m-14y,MU+FU}$
4- $S_{6y-14y} - I_{9m-6y,MU+FU}$
5- $I_{9m-14y,MU+FU}$
6- $S_{9m-6y,FU}$
7- $S_{9m-6y,MU+FU}$
8- $I_{9m-6y,MU+FU}$

a-Additional cost of 20% for mobile units

Table 5 Degree of membership of technical and cost constrainqs for each strategy

Strategy	Technical constraint	Cost constraint	Min (b)
1	.20	0.467	.20
2	.45	0.00	0.00
3	.30	.930	.30
4	.30	.540	.30
5	.20	.206	.20
6	.60	.948	.60
7	.50	.938	.50
8	.40	.437	.40

Now we have all the components of the decision model: a set A of possible actions the eight possible strategies

a set of 'goals' $G_i(i \in N)$ defined on A : the relative efficacy of each possible strategy (third column of Table 4);

a set of constraints $C_j(j \in M)$ defined on A : the minimum between the technical and costs constraints (last column of Table 5);

The fuzzy decision D , that simultaneously satisfies the given goals Gi and Cj constraints is than

$$D(a) = \min[G_i(a), C_j(a)]$$

For all 'a' belongs to A that is as shown in Table 6

Therefore, the strategy that has the maximum degree of membership in the set of decision in strategy number 6,which selectively vaccinates children aged 9 months 6 years, using only fixed units of the health system .

Table 6 Fuzzy decision setting

strategy	Gi(a)	Cj(a)	D(a)
1	0.049	0.200	0.049
2	0.098	0.00	0.00
3	1.000	0.300	0.300
4	.0127	0.300	0.127
5	0.045	0.200	0.045
6	0.770	0.600	0.600
7	0.761	0.500	0.500
8	0.147	0.400	0.147

4. CONCLUSION

We think that the Fuzzy Logic approach for designing the control strategy against the recent measles epidemic in Allahabad was very useful in the sense that it allowed the combination of intuitive information from public health experts and cost constraints into a coherent model. Moreover it proved to be very effective, in the sense that the strategy adopted resulted in significant control of the epidemic. Our results, notwithstanding several intervening factors outside our control during the implementation of the proposed strategy, are very encouraging in demonstrating the potential of new techniques for the design of interventions in public health.

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